Geometrization of matter and antimatter through coadjoint action of a group on its momentum space

1: Charges as additional scalar components of the momentum of a group acting on a 10d-space. Geometrical definition of antimatter.

Jean-Pierre Petit & Pierre Midy

Marseille Observatory, France
jppetit1937@yahoo.fr

Abstract

Through a new four components non-connex group, acting on a ten dimensional space, composed by (x,y,z,t) plus six additional dimensions we give a description of particles like photon, proton, neutron, electrons, neutrinos (e, μ and τ) and their anti, through the coadjoint action on the momentum space. Quantum numbers become components of the moments. Matter and antimatter are interpreted as two different movements of mass-points in this

$$\{ \zeta^1, \zeta^2, \zeta^3, \zeta^4, \zeta^5, \zeta^6, x , y , z , t \} \text{ space}$$

matter movement taking place in the $\{\zeta^i > 0\}$ half space and antimatter in the remnant $\{\zeta^i < 0\}$ one.

The ζ-Symmetry :

$$\{\zeta^i \dashrightarrow - \zeta^i \}$$

which there goes with charge conjugation, becomes the definition of matter-antimatter duality.

1- Introduction

As pointed out by J.M.Souriau in his book [1] the Poincaré group, as a dynamic group for physics, arises a problem about the sign of the mass.

Everything starts from the Lorentz group L, whose element **L** is axiomaticaly defined by:

(1)

$$\bar{\mathbf{L}} \ \mathbf{G} \ \mathbf{L} = \mathbf{G}$$

where:
(2)

$$\mathbf{G} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The Lorentz group acts on space-time:
(3)

$$\boldsymbol{\xi} = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

through the action:

(4)

$$\mathbf{L}\ \boldsymbol{\xi}$$

The matrix **G** comes from the expression of the Lorentz metric (with c=1):

(5)

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = \overline{d\boldsymbol{\xi}}\ \mathbf{G}\ d\boldsymbol{\xi}$$

We know than the Lorentz group is composed by four components :

L_n is the neutral componant, which contains the neutral element **1**, i.e. the peculiar matrix:

(6)

$$\mathbf{A}_n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

L_s , the second component, contains the matrix :

(7)

$$\mathbf{A}_s = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which reverses space.

L_t , the third component, contains the matrix :

(8)

$$\mathbf{A}_t = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

which reverses time.

L_{st} , the fourth component, contains the matrix :

(9)

$$\mathbf{A}_{st} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

which reverses both space and time.

From the Lorentz group one builds the Poincaré group G_p, whose element is:

(10)

$$g_p = \begin{pmatrix} \mathbf{L} & \mathbf{C} \\ \mathbf{0} & 1 \end{pmatrix}$$

C is a space-time translation:

(11)

$$\mathbf{C} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta t \end{pmatrix}$$

If we use the four components of the complete Lorentz group L , (10) will be called the complete Poincaré group. As the Lotentz group, it owns four components:

- Its neutral component:

(12)

$$g_{p_n} = \begin{pmatrix} \mathbf{L}_n & \mathbf{C} \\ \mathbf{0} & 1 \end{pmatrix}$$

built with the neutral component L_n of the Lorentz group L.

- A second component:

(13)

$$g_{p_s} = \begin{pmatrix} L_s & C \\ 0 & 1 \end{pmatrix}$$

built with the component L_s of the Lorentz group.

- A third component:

(14)

$$g_{p_t} = \begin{pmatrix} L_t & C \\ 0 & 1 \end{pmatrix}$$

built with the component L_t of the Lorentz group.

- and a fourth one:

(15)

$$g_{p_{st}} = \begin{pmatrix} L_{st} & C \\ 0 & 1 \end{pmatrix}$$

built with the component L_{st} of the Lorentz group.

A group acts on its momentum space [1]. Call J_p the momentum space associated to the Poincaré group.

Each peculiar moment $\mathbf{J}_p \in J_p$, is a peculiar movement of the relativistic mass point, described by this group. On may compute the caodjoint action of the group on the momentum [1].

The momentum is a set of 10 components (equal to the dimension of the group). These components are:

(16)

$$\mathbf{J}\mathrm{p} = \{ E , p_x , p_y , p_z , f_x , f_y , f_z , s_x , s_y , s_z \} = \{ E , \mathbf{p} , \mathbf{f} , \mathbf{s} \}$$

E is the energy.
p is the impulsion vector:

(17)

$$\mathbf{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

f is the passage vector [1].

(18)

$$\mathbf{f} = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}$$

s is an antisymmetric (3,3) matrix, whose independant components are
(19)

$$\{ s_x , s_y , s_z \}$$

The momentum can be arranged into a matrix form [1], with:

(20)

$$\mathbf{s} = \begin{pmatrix} 0 & -s_z & s_y \\ s_z & 0 & -s_x \\ -s_y & s_x & 0 \end{pmatrix}$$

and:

(21)

$$\mathbf{M} = \begin{pmatrix} \mathbf{s} & \mathbf{f} \\ -\overline{\mathbf{f}} & 0 \end{pmatrix}$$

Introduce the impulsion-Energy four-vector:

(22)

$$\mathbf{P} = \begin{pmatrix} p_x \\ p_y \\ p_z \\ E \end{pmatrix}$$

(23)

$$\mathbf{J_p} = \begin{pmatrix} 0 & -s_z & s_y & f_x & -p_x \\ s_z & 0 & -s_x & f_y & -p_y \\ -s_y & s_x & 0 & f_z & -p_z \\ -f_x & -f_y & -f_z & 0 & -E \\ p_x & p_y & p_z & E & 0 \end{pmatrix}$$

or:

(24)

$$J_p = \begin{pmatrix} \mathbf{M} & -\mathbf{P} \\ \overline{\mathbf{P}} & 0 \end{pmatrix}$$

Then the coadjoint action of the Poincaré group can be written into a matrix form:

(25)

$$J'_p = g_p \, J_p \, \bar{g}_p$$

More explicitely:
(26)

$$\mathbf{M}' = \mathbf{L}\,\mathbf{M}\,\overline{\mathbf{L}} + (\,\mathbf{C}\,\overline{\mathbf{P}}\,\overline{\mathbf{L}} - \mathbf{L}\,\mathbf{P}\,\overline{\mathbf{C}}\,)$$

$$\mathbf{P}' = \mathbf{L}\,\mathbf{P}$$

It is interesting to study the impact of the different components of the complete Poincaré group on the components of its momentum. We can concentrate on peculiar matrixes :

(27)

$$\mathbf{B} = \begin{pmatrix} \boldsymbol{\lambda} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mu & 0 \\ \mathbf{0} & 0 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{A} & 0 \\ 0 & 1 \end{pmatrix} \quad \text{with} \quad \mathbf{A} = \begin{pmatrix} \boldsymbol{\lambda} & \mathbf{0} \\ \mathbf{0} & \mu \end{pmatrix}$$

A is the associated Lorentz matrix.

The coadjoint action gives:

(28)

$$\begin{pmatrix} \mathbf{p}' \\ E' \end{pmatrix} = \begin{pmatrix} \boldsymbol{\lambda}\,\mathbf{p} \\ \mu E \end{pmatrix}$$

(29)

$$\begin{pmatrix} \mathbf{s}' & \mathbf{f}' \\ -\overline{\mathbf{f}}' & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{s} & \boldsymbol{\lambda}\,\mu\,\mathbf{f} \\ -\boldsymbol{\lambda}\,\mu\,\mathbf{f} & 0 \end{pmatrix}$$

g_{p_n} contains the element $\mathbf{B}_n = \mathbf{B}(\,\mathbf{1}\,,\,1\,)$

and is the neutral component of the complet Poincaré group.

The corresponding coadjoint action is:

E --> E ; **p** --> **p** ; **f** ---> **f** ; **s** ----> **s**

g_{p_s} contains the element $B_s = B(-1, 1)$

which reverses space. The corresponding coadjoint action is:

E --> E ; **p** --> - **p** ; **f** ---> - **f** ; **s** ----> **s**

g_{p_t} = contains the element $B_t = B(1, -1)$

which reverses time. The corresponding coadjoint action is:

E --> - E ; **p** --> **p** ; **f** ---> - **f** ; **s** ----> **s**

$g_{p_{st}}$ = contains the element $B_{st} = B(1, -1)$

which reverses both space and time. The corresponding coadjoint action is:

E --> - E ; **p** --> - **p** ; **f** ---> **f** ; **s** ----> **s**

As pointed out by J.M.Souriau [1], The two components

$$g_{p_t} \quad \text{and} \quad g_{p_{st}}$$

go with the inversion of the energy E ----> - E , so that it implies the inversion of the mass m ---> - m

Define the following sets of matrixes:

(30)

$$g_{p_n} \in G_n \qquad g_{p_s} \in G_s \qquad g_{p_t} \in G_t \qquad g_{p_{st}} \in G_{st} \quad .$$

The complete Poincaré group is:

(31)

$$G_p = G_n \cup G_s \cup G_t \cup G_{st}$$

The neutral component Gn is the first sub-group. The orthochron group [1]:

(32)

$$G_o = G_n \cup G_s$$

is also a sub-group of the Poincaré group.

The antichron part of the group [1]:

(33)

$G_{ac} = G_t \cup G_{st}$ is not a group. Obvioulsy:

(34)

$$G_p = G_o \cup G_{ac}$$

As pointed out in [1] the presence of the elements of $G_{ac} = G_t \cup G_{st}$ may produce negative mass particles, as peculiar movements of matter, runing backward in time. In his book [1] J.M.Souriau suggests two solution:

- Either one simply decides that negative masses cannot exist.
- Either the Poincaré group is limited to its orthochron subgroup.

(35)

$$G_o = G_n \cup G_s$$

2- The central extension of the Poincaré group

(36)

$$g_{pe} = \begin{pmatrix} 1 & \mathbf{0} & \phi \\ \mathbf{0} & L_o & C \\ 0 & \mathbf{0} & 1 \end{pmatrix}$$

is the central extension of the Poincaré group, built from the orhochron sub-group.
The corresponding action is:
(37)

$$g_{pe} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{0} & \phi \\ \mathbf{0} & L_o & C \\ 0 & \mathbf{0} & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix}$$

ζ is an additional dimension, a fifth dimension. The dimension of the group becomes 11 and the momentum gets a corresponding extra component:

(38)

$$\mathbf{J}_{pe} = \{ \chi , \mathbf{M} , \mathbf{P} \} = \{ \chi , \mathbf{J}_p \}$$

The coadjoint action gives:
(39)

$$\chi' = \chi$$

$$P' = L P$$

$$M' = L M \bar{L} + (C \bar{P} \bar{L} - L P \bar{C})$$

The physical meaning of this 11th component χ was neven clearly undestood. Through his geometric quantification method, J.M.Souriau shows than the spin must be quanticized [1]. Choosing a coordinate system in which the passage becomes zero, and considering only z-motions, the $\mathbf{J}_p$ the momentum matrix becomes:

(40)

$$J_p = \begin{pmatrix} 0 & -s & 0 & 0 & 0 \\ s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -p \\ 0 & 0 & 0 & 0 & -E \\ 0 & 0 & p & E & 0 \end{pmatrix}$$

where E is the energy, p the modulus of the vector impulsion and s the spin.

Photons correspond to
(41)

$$J_p = \begin{pmatrix} 0 & -\hbar & 0 & 0 & 0 \\ \hbar & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{E}{c} \\ 0 & 0 & 0 & 0 & -E \\ 0 & 0 & \frac{E}{c} & E & 0 \end{pmatrix} \quad J_p = \begin{pmatrix} 0 & \hbar & 0 & 0 & 0 \\ -\hbar & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{E}{c} \\ 0 & 0 & 0 & 0 & -E \\ 0 & 0 & \frac{E}{c} & E & 0 \end{pmatrix}$$

right left

with two distinct helicities: right and left (polarization).

Neutrinos correspond to:
(42)

$$J_p = \begin{pmatrix} 0 & -\frac{\hbar}{2} & 0 & 0 & 0 \\ \frac{\hbar}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{E}{c} \\ 0 & 0 & 0 & 0 & -E \\ 0 & 0 & \frac{E}{c} & E & 0 \end{pmatrix} \quad J_p = \begin{pmatrix} 0 & \frac{\hbar}{2} & 0 & 0 & 0 \\ -\frac{\hbar}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{E}{c} \\ 0 & 0 & 0 & 0 & -E \\ 0 & 0 & \frac{E}{c} & E & 0 \end{pmatrix}$$

right left

with also two distinct helicities.

Non zero mass particles like proton, electron, neutron, correspond to:
(43)

$$J_p = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\hbar}{2} & 0 & 0 \\ 0 & \frac{\hbar}{2} & 0 & 0 & -p \\ 0 & 0 & 0 & 0 & E \\ 0 & 0 & p & -E & 0 \end{pmatrix}$$

with:
(44)

$$E = \frac{m c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

(45)

$$p = \frac{m v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

From the extended Poincaré group (36), through the Kostant-Kirilov-Souriau method one can derive [1] the relativistic quantum Klein-Gordon equation. Similarly [1] the non-relativist Bargmann group (1960) gives the non-relativistic Schödinger equation.

What about antimatter?

In a former book [2] J.M. Souriau developped general relativity in five dimensions, adding an extra dimension ζ to space-time (x, y, z, t)

Then, reference [2], Chater VII, page 413, he identifies the inversion of the fifth dimension ($\zeta \longrightarrow -\zeta$) to the charge conjugation (or charge inversion, or C-symmetry) transforming matter into antimatter.

3- A description of quantum numbers as components of the moment of an extended group

The Poincaré group can be extended as many times one wants. Let us do it six times. Then we get:

(46)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & \phi_1 \\ 0 & 1 & 0 & 0 & 0 & 0 & \mathbf{0} & \phi_2 \\ 0 & 0 & 1 & 0 & 0 & 0 & \mathbf{0} & \phi_3 \\ 0 & 0 & 0 & 1 & 0 & 0 & \mathbf{0} & \phi_4 \\ 0 & 0 & 0 & 0 & 1 & 0 & \mathbf{0} & \phi_5 \\ 0 & 0 & 0 & 0 & 0 & 1 & \mathbf{0} & \phi_6 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & L_o & \mathbf{C} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta_1 + \phi_1 \\ \zeta_2 + \phi_2 \\ \zeta_3 + \phi_3 \\ \zeta_4 + \phi_4 \\ \zeta_5 + \phi_5 \\ \zeta_6 + \phi_6 \\ L_o \xi + \mathbf{C} \\ 1 \end{pmatrix}$$

This two components group (due to the two components of the orthochron Lorentz group Lo) acts on a ten dimensional space:

$$\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , \zeta^6 , x , y , z , t \}$$

i.e. space time (x , y , z , t)

plus six additional dimensions

$$\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , \zeta^6 \}$$

The momentum becomes:
(47)

$$\mathbf{J}_{pe} = \{ \chi^1, \chi^2, \chi^3, \chi^4, \chi^5, \chi^6, \mathbf{J}_p \}$$

where $\mathbf{J}_p$ represent the classical expression of the Poincaré group's momentum.

The coadjoint action is:
(48)

$$\chi^{i\,'} = \chi^i \quad \text{for } i = 1 \text{ to } 6$$

$$\mathbf{P}' = \mathbf{L}\,\mathbf{P}$$

$$\mathbf{M}' = \mathbf{L}\,\mathbf{M}\,\overline{\mathbf{L}} + (\,\mathbf{C}\,\overline{\mathbf{P}}\,\overline{\mathbf{L}} - \mathbf{L}\,\mathbf{P}\,\overline{\mathbf{C}}\,)$$

All these additional scalars are conserved and we identify these to the following classical quantum numbers:

(49)

$\chi^1 = q$ (electric charge)

$\chi^2 = c_B$ (baryonic charge)

$\chi^3 = c_L$ (leptonic charge)

$\chi^4 = c_\mu$ (muonic charge)

$\chi^5 = c_\tau$ (tauonic charge)

$\chi^6 = \varpi$ (gyromagnetic coefficient)

We give to each first five numbers three possible values : { -1 , 0 , +1 }

The value of the gyromagnetic factor ϖ depend of the considered particle.

The momentum space is supposed to be a continuum, but one assume that discrete values of some components correspond to real particules, from physics' world. Then we get a description of elementary particles in terms of group's orbits. We can write the momentum:

(50) $\mathbf{J}_{pe} = \{ q , c_B , c_L , c_\mu , c_\tau , \varpi , \mathbf{J}_p \}$

$\mathbf{J}_\phi = \{ 0 , 0 , 0 , 0 , 0 , 0 , \mathbf{J}_p \}$: photon

$\mathbf{J}_p = \{ 1 , 1 , 0 , 0 , 0 , \varpi_p , \mathbf{J}_p \}$: proton

$\mathbf{J}_{\overline{p}} = \{ -1 , -1 , 0 , 0 , 0 , -\varpi_p , \mathbf{J}_p \}$: anti proton

$\mathbf{J}_n = \{ 0 , 1 , 0 , 0 , 0 , \varpi_n , \mathbf{J}_p \}$: neutron

$\mathbf{J}_{\overline{n}} = \{ 0 , -1 , 0 , 0 , 0 , -\varpi_n , \mathbf{J}_p \}$: anti neutron

$\mathbf{J}_e = \{ -1 , 0 , 1 , 0 , 0 , \varpi_e , \mathbf{J}_p \}$: electron

$J_{\bar{e}} = \{ 1 , 0 , -1 , 0 , 0 , -\varpi_e , J_p \}$: anti electron

$J_{\nu e} = \{ 0 , 0 , 1 , 0 , 0 , \varpi_{\nu e} , J_p \}$: electronic neutrino

$J_{\bar{\nu}_e} = \{ 0 , 0 , -1 , 0 , 0 , -\varpi_{\nu e} , J_p \}$: anti e – neutrino

$J_{\nu\mu} = \{ 0 , 0 , 0 , 1 , 0 , \varpi_{\nu\mu} , J_p \}$: μ neutrino

$J_{\bar{\nu}_\mu} = \{ 0 , 0 , 0 , -1 , 0 , -\varpi_{\nu\mu} , J_p \}$: anti μ neutrino

$J_{\nu\tau} = \{ 0 , 0 , 0 , 0 , 1 , \varpi_{\nu\tau} , J_p \}$: τ neutrino

$J_{\bar{\nu}_\tau} = \{ 0 , 0 , 0 , 0 , -1 , -\varpi_{\nu\tau} , J_p \}$: anti τ neutrino

We transform a particle into the corresponding antiparticle through charge conjugation (C-symmetry). The charges of the photon are all zero, so that it identifies with its antiparticle.

4- Suggested geometric definition of antimatter

A particle is a species, corresponding to a sub-set of the momentum space. It corresponds to peculiar choices in some components of the momentum, the charges:

(51)

$$\{ q , c_B , c_L , c_\mu , c_\tau , \varpi \}$$

A momentum is a movement of a mass-point, governed by a dynamic group. Here an extension of the orthochron Poincaré's sub-group.

Classically (Dirac's antimatter) one considers that reversing the charge (C-symmetry of charge conjugation) transforms matter into anti matter

(52)

$$\{ - q , - c_B , - c_L , - c_\mu , - c_\tau , - \varpi \}$$

Then we can classify the particles, through their momentum space, into two sub-sets, the first containing matter and the second anti matter. Schematically, photons have been figured on the borde between the two, for they are identical to antiphotons. See figure 1.

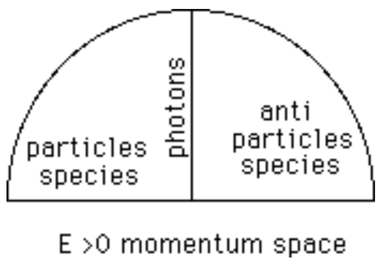

Fig.1: Classification of particles.

As we know each momentum correspond to a movement. Here we consider movements in a ten-dimensional space, a fibered space-time, as evoked on figure 2.

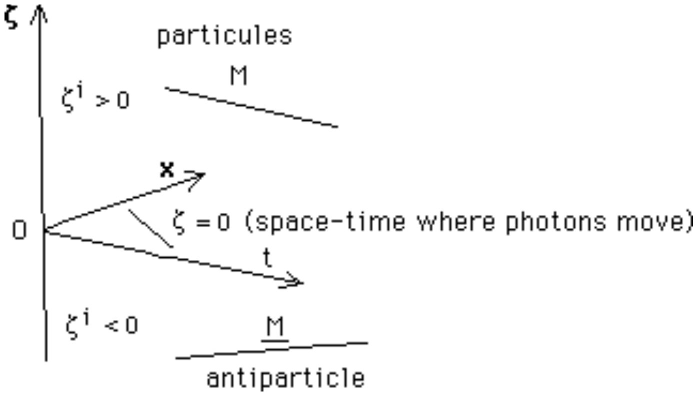

Fig.2: **Fibered space-time.**

As shown of the figure we suggest that matter-antimatter duality corresponds to a:
(53)

$$\zeta \text{ - Symmetry: } \{\zeta^{\,i}\} \text{ ---> } \{ -\zeta^{\,i} \}$$

Particles move in $\{ \zeta^{\,i} > 0 \}$ half-space and antiparticles in the other $\{ \zeta^{\,i} < 0 \}$ one. Photons move in $\{ \zeta^{\,i} = 0 \}$ plane. Their movement is not changed by ζ-Symmetry, so that they are identical to their antiparticle.

In this paper we deal with an extended 16-dimensional orthochron group. We can figure schematically the coadjoint action of such a group on its moment space and associated movement space. See figures 3, 4 and 5.

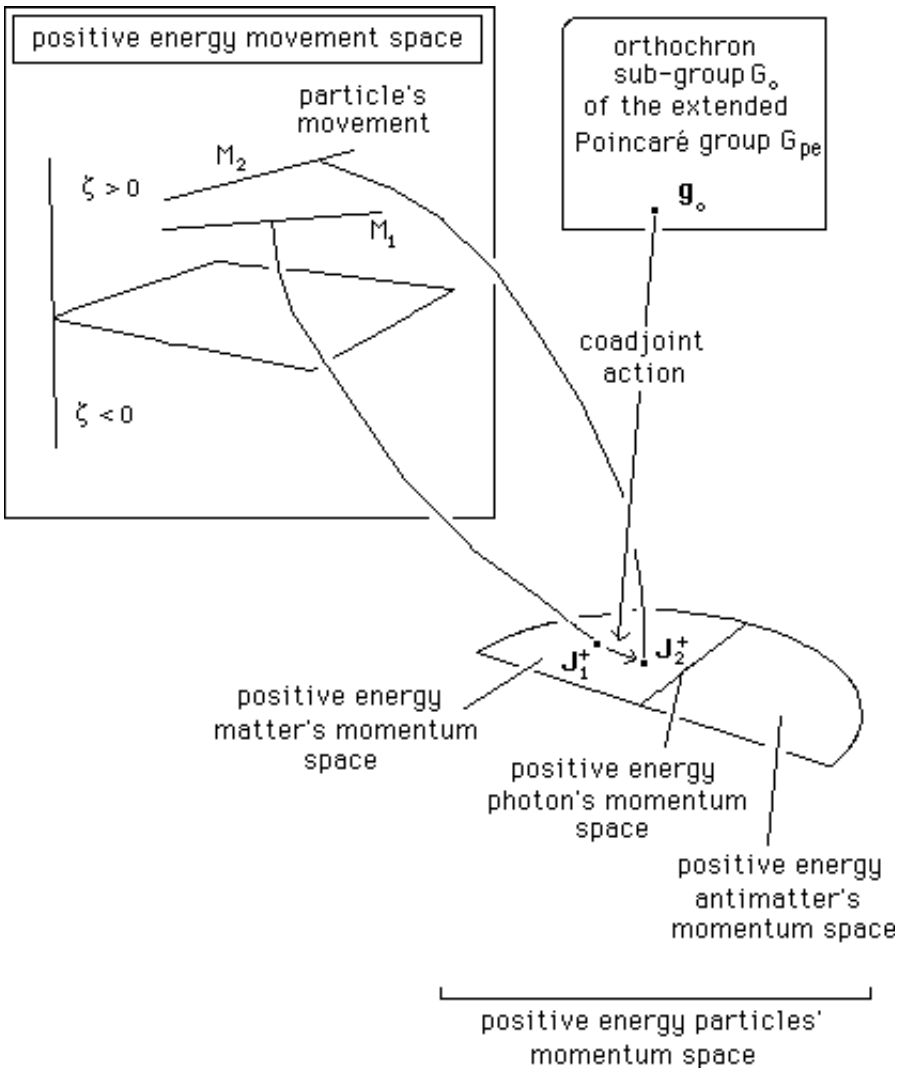

Fig. 3 : **Movement of matter, in the $\{ \zeta^{\,i} > 0 \}$ half 10d-space and coadjoint action on the momentum. The link between momentum and movement has been figured.**

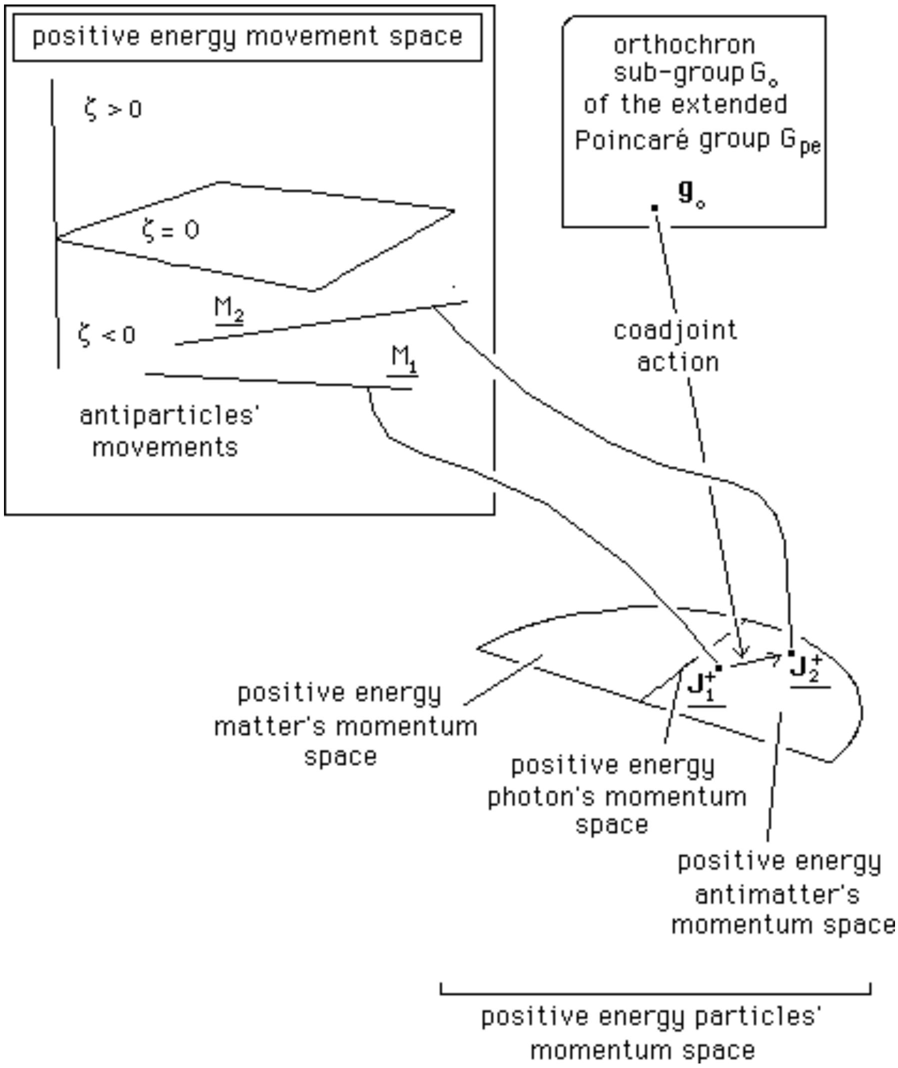

Fig. 4 : **Movement of antimatter, in the** $\{ \zeta^i < 0 \}$ **half 10d-space and coadjoint action on the momentum. The link between momentum and movement has been figured.**

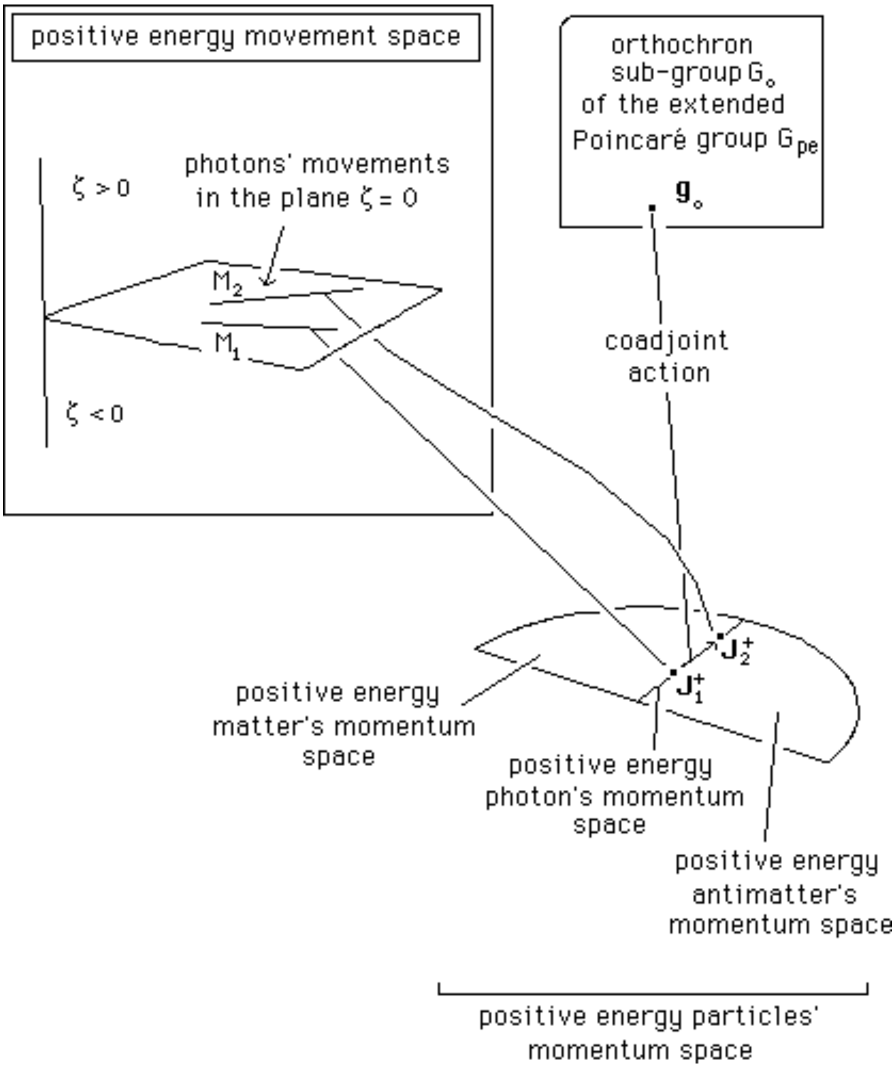

Fig. 5: Movement of photons, in the { $\zeta^i = 0$ } plane and coadjoint action on the momentum. The link between momentum and movement has been figured.

Conclusion

We have extended the othochron Poincaré sub-group, corresponding to positive energy particles to a 16-dimensional group, acting:

- On a 16-dimensional momentum space

- On a 10-dimensional movement space.

The extension gives the momentum six extra components, which are identified to charges, so that we get a geometric description of usual elementary particles: photon, proton, electron, neutrons , e , μ and τ neutrinos and their antis.

This provides a classification of particles in terms of momentum's components, defining three basic species:

- Particles
- Antiparticles
- Photons.

each corresponding to a sub-set of the ($E > 0$) momentum space. Then we suggest a basic definition of antimatter, and photons, in terms of peculiar movements in a 10d-space.

$\{ \zeta^i > 0 \}$ corresponding to matter.

$\{ \zeta^i < 0 \}$ corresponding to antimatter.

$\{ \zeta^i = 0 \}$ corresponding to photons.

This is similar to Plato's vision.

The objects move in a 10-dimensional space, but the inhabitants of the cavern can just see the 4-dimensional (x,y,z,t) shadows of these movements.

References

[1] J.M. Souriau: "Structure des Systèmes Dynamiques", Dunod, Ed. 1972 (in French); *tr.* "Structures of Dynamical Systems", Birkhauser, 1997. ISBN 0817636951.
[2] J.M. Souriau: Géométrie et relativité. Ed. Hermann-France, 1964.
[3] P.M. Dirac: "A theory of protons and electrons", Dec. 6th 1929, published in *Proceedings of Royal Society* (London), 1930: A **126**, 360–365.

Acknowledgements

This work was supported by french CNRS and Brevets et Développements Dreyer company, France.
Déposé sous pli cacheté à l'Académie des Sciences de Paris, 1998.
© French Academy of Sciences, Paris, 1998.